turbulent. It was assumed that in this case, the aft microphone gave a more realistic representation of the acoustic environment. In addition, it was assumed that the disturbance function Σ was comprised solelv of acoustic disturbances. A plot of transition data obtained in the AEDC 16T as a function of acoustic level is shown in Fig. 3. The theoretical line was calculated for a Mach number of 0.8. Since the range of Mach numbers is small (0.3–1.6) and since the effect of Mach number is small for this range, this line represents a "mean" prediction line.

Figures 2 and 3 indicate that transition can be predicted to within 10% with reasonable consistency. Additional factors having a significant effect on the data which are not included in the model and/or whose effect could not be removed from the data are: 1) pitch and yaw misalignment; and 2) acoustic levels due to proximity of transition to microphone location.

The present technique has been applied to the low subsonic and supersonic flow regimes for the data of Refs. 4–6 and of 12 and 13. The preliminary results indicate that an extension to these flow regimes is feasible.

Conclusions

The consistency of the preceding results shows that the proposed technique for the prediction of beginning of transition location is justified, at least for 10° included-angle cones. Extension of this method to geometries involving strong pressure gradients may well await a more detailed understanding of the transition process. Therefore, the present work is not presented as a final solution but merely as a useful tool to be used in lieu of exact analysis. The following conclusions can be drawn from the present study. 1) Beginning of transition on 10° included-angle cones can be predicted to within 10%. 2) The technique appears to be extendible to supersonic and low subsonic flow regimes. 3) Noise appears to be at least as important as vorticity in causing transition and in many cases is more important.

References

[1] Liepmann, H. W., "Investigation of Boundary-Layer Transition on Concave Walls," ARC 4J28, 1945, NACA.

[2] Hairston, D. E., "Survey and Evaluation of Current Boundary-Layer Transition Prediction Techniques," AIAA Paper 71-985, Washington, D.C., 1971.

[3] Van Driest, E. R. and Blumer, C. B., "Boundary-Layer Transition: Free Stream Turbulence and Pressure Gradient Effects," *AIAA Journal*, Vol. 1, No. 6, June 1963, pp. 1303–1307.

[4] Schubauer, G. B. and Skramstad, H. F., "Laminar Boundary-Layer Oscillations and Transition on a Flat Plate," Rept. 909, 1948, NACA.

[5] Wells, C. S., Jr., "Effects of Freestream Turbulence on Boundary-Layer Transition," *AIAA Journal*, Vol. 5, No. 1, Jan. 1967, pp. 172–175.

[6] Pate, S. R., "Measurements and Correlations of Transition Reynolds Number on Sharp Slender Cones at High Speed," AEDC-TR-69-172, Dec. 1969, Arnold Engineering Development Center, Tullahoma, Tenn.

[7] Credle, O. P., "An Evaluation of the Fluctuating Airframe Environment in the AEDC-PWT 4-ft Transonic Tunnel," AEDC-TR-69-236, Nov. 1969, Arnold Engineering Development Center, Tullahoma, Tenn.

[8] Credle, O. P. and Carleton, W. E., "Determination of Transition Reynolds Numbers in the Transonic Mach Number Range," AEDC-TR-70-218, Oct. 1970, Arnold Engineering Development Center, Tullahoma, Tenn.

[9] Credle, O. P. and Shadow, T. O., "Evaluation of the Overall Root-Mean-Square Fluctuating Pressure Levels in AEDC PWT 16-ft Transonic Tunnel," AEDC-TR-70-7, Feb. 1970, Arnold Engineering Development Center, Tullahoma, Tenn.

[10] Taylor, G. I., "Statistical Theory of Turbulence, Part V," *Proceedings of the Royal Society (London)*, Vol. A156, 1936, pp. 307–317.

[11] Hinze, J. O., *Turbulence: An Introduction to Its Mechanism and Theory*, McGraw-Hill, New York, 1959.

[12] Palko, R. L., Burt, R. H., and Ray, A. D., "An Experimental Investigation of Boundary-Layer Transition on Flat Plates at Mach Numbers 5, 8, and 10," AEDC-TDR-64-167, Aug. 1964, Arnold Engineering Development Center, Tullahoma, Tenn.

[13] Benek, J. A. and High, M. D., "A Method for the Prediction of the Effects of Free-Stream Disturbances on Boundary-Layer Transition," AEDC-TR-73-158, Oct. 1973, Arnold Engineering Development Center, Tullahoma, Tenn.

Note on Unsteady Boundary-Layer Separation

JAMES C. WILLIAMS III* AND W. DONALD JOHNSON†
North Carolina State University, Raleigh, N.C.

Nomenclature

A, B = constants
f = dimensionless stream function
t = time
u = x component of velocity
u_δ = velocity at upper edge of boundary layer
U = velocity of moving coordinate system
U_∞ = uniform "freestream" velocity
v = y component of velocity
x = physical coordinate parallel to body surface
x_s = location of the separation point
y = physical coordinate normal to body surface
η = dimensionless coordinate normal to body surface
ν = kinematic viscosity
ξ = dimensionless coordinate parallel to body surface
ψ = stream function

Superscript
"–" = overbars denote quantities in moving coordinate system

Introduction

It was recognized quite early that the vanishing of the shear at the wall, which has been such a useful criterion for steady boundary-layer separation, cannot be taken as representing unsteady boundary-layer separation.[1–3] Moore,[1] Rott,[4] and Sears[3] have formulated a model for unsteady separation; a model in which the unsteady separation point is characterized by the vanishing of both the velocity and the shear at some point in the boundary layer ". . . in a flow seen by an observer moving with the separation point."[3] Moore[1] and Sears and Telionis[5] also argue that this separation point is characterized by a singularity in the solution to the boundary-layer equations.

In the development of this model for unsteady separation both Moore[1] and Telionis[6] noted a relationship between unsteady boundary layers and steady boundary layers over a moving wall. Moore presented intuitive arguments to relate unsteady separation to steady separation over a moving wall and, on the basis of these arguments, concluded that "a criterion of simultaneous vanishing of shear and velocity is the proper generalization of the usual definition of steady separation, for the case of a separation point moving slowly along a surface." Telionis, also recognized the relation between unsteady separation and steady separation over a moving wall and further showed that a given flow over a moving wall can be transformed into an equivalent unsteady problem by a simple transformation. The analysis of Telionis involved only a transformation of velocities, however,

Received February 8, 1974; revision received April 4, 1974. This work was supported by the U.S. Army Research Office – Durham under Grant DA-ARO-D-31-124-72-G134.

Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Nonsteady Aerodynamics.

* Professor and Associate Head, Department of Mechanical and Aerospace Engineering. Associate Fellow AIAA.

† Graduate Research Assistant, Department of Mechanical and Aerospace Engineering. Member AIAA.

not a transformation between moving and stationary coordinate systems. Such a transformation is necessary to relate the moving (unsteady) separation point in a stationary coordinate system to a stationary separation viewed in a moving coordinate system.

The relationship between unsteady boundary-layer separation and steady separation over a moving wall has always been an intuitive one. As far as the authors have been able to determine, it has not been possible, up to this point, to transform a given unsteady flow into a steady flow over a moving wall, study the separation over the moving wall, and relate this back to the unsteady flow.

The purpose of the present Note is to show that for a certain class of unsteady external streams the unsteady boundary-layer flow over a stationary wall may be transformed into a steady boundary-layer flow over a wall *moving with the speed of the separation point*. Thus, the link between the unsteady flow and the flow over a moving wall is established in this case. The solution to the equivalent steady problem may be obtained using conventional techniques and the troublesome problem of attempting to integrate "upstream" in the flow is avoided. The solution of a simple problem, obtained by this technique, verifies the characteristics of the unsteady separation point postulated in the Moore-Rott-Sears model.

Analysis

We consider the general two-dimensional unsteady boundary-layer problem in an incompressible, constant property fluid. The boundary-layer equations and boundary conditions for this problem are

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_\delta}{\partial t} + u_\delta\frac{\partial u_\delta}{\partial x} + v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u(x,0,t) = v(x,0,t) = 0 \quad \lim_{y\to\infty} u(x,y,t) = u_\delta(x,t) \tag{3}$$

We wish however to examine this flow in a coordinate system which is moving parallel to the x axis with a velocity $U(t)$. The relationships between the moving coordinates and the velocities in the fixed coordinate system are

$$x = \bar{x} + \int U(t)dt, \quad y = \bar{y}, \quad t = \bar{t}, \quad u = \bar{u} + U, \quad v = \bar{v}$$

where the overbars refer to the moving coordinate system. In the moving coordinate system the boundary-layer equations and the corresponding boundary conditions become

$$\partial\bar{u}/\partial\bar{x} + \partial\bar{v}/\partial\bar{y} = 0 \tag{4}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial \bar{u}_\delta}{\partial \bar{t}} + \bar{u}_\delta\frac{\partial \bar{u}_\delta}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \tag{5}$$

$$\bar{u}(\bar{x},0,\bar{t}) = -U, \quad \bar{v}(\bar{x},0,\bar{t}) = 0 \tag{6}$$

$$\lim_{y\to\infty} \bar{u}(\bar{x},\bar{y},\bar{t}) = u_\delta(x,t) - U(t) = \bar{u}_\delta(\bar{x},\bar{t})$$

We now note that if the external velocity distribution in the fixed coordinate system is a function only of a linear combination of x and t, i.e., $u_\delta(x,t) = f(Ax+Bt)$ and if the relative velocity between the two coordinate systems $[U(t)]$ is constant, the external velocity distribution in the moving coordinate system is given by

$$\bar{u}_\delta(\bar{x},\bar{t}) = f(A\bar{x} + AU\bar{t} + B\bar{t}) - U \tag{7}$$

If the relative velocity U is taken as $-B/A$ we obtain

$$\bar{u}_\delta(\bar{x},\bar{t}) = f(A\bar{x}) + B/A \tag{8}$$

and

$$\bar{u}(\bar{x},0,\bar{t}) = +B/A \tag{9}$$

Thus in the moving coordinate system the external velocity is independent of time and the wall boundary condition is constant so that the abovementioned problem is steady. If separation occurs, an observer in the moving coordinate system will observe separation to occur at some fixed point, say $\bar{x}_s$, while an observer in the fixed coordinate system will see a moving separation point whose x location is given by $x_s = \bar{x}_s - (B/A)t$. The moving coordinate system in this case is then the coordinate system which moves with the separation point. It should be possible, using the preceding analysis, to obtain some additional insight into the phenomenon of unsteady separation.

The preceding problem can be handled by conventional techniques for solving steady nonsimilar boundary-layer problems. If we introduce the stream function, a dimensionless $\bar{y}$ coordinate and a dimensionless $\bar{x}$ coordinate defined, respectively, by

$$\psi(\bar{x},\bar{y}) = (\bar{u}_\delta \bar{x} v)^{1/2} f(\eta,\xi), \quad \eta = \bar{y}(\bar{u}_\delta/v\bar{x})^{1/2}, \quad \xi = \bar{x}A$$

the momentum equation becomes

$$f''' + \frac{1+M}{2}ff'' + M(1-f'^2) = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \tag{10}$$

where

$$M = (\bar{x}/\bar{u}_\delta)\, d\bar{u}_\delta/d\bar{x}$$

The boundary conditions which are to be used in solving Eq. (10) are

$$f(\xi,0) = 0, \quad f'(\xi,0) = (B/A\bar{u}_\delta) \quad \lim_{\eta\to\infty} f'(\xi,\eta) = 1 \tag{11}$$

Equation (10) is a standard form for the steady nonsimilar laminar boundary-layer equation which may be solved using an implicit finite-difference technique similar to that outlined by Blottner.[7] The moving-wall boundary condition given in Eq. (11) is easily incorporated into this technique.

Solution for a Simple Flow

As an example we have considered an unsteady variation of Howarth's linearly retarded flow for which the unsteady velocity in the external stream is given by

$$u_\delta(x,t) = U_\infty(1 - Ax - Bt)$$

The steady flow equivalent is just the steady form of Howarth's retarded flow over a wall moving with velocity $+B/A$ (as seen by an observer moving with velocity $-B/A$ relative to the fixed wall). Solutions were obtained for various values of the dimensionless wall velocity B/AU_∞. Since each of these solutions exhibited the same general characteristics, only the solution for $B/AU_\infty = 0.2$ will be discussed here.

Since the steady flow, as seen in the moving coordinate system, is equivalent to steady flow over a moving wall, the results obtained in this coordinate system should and do agree with the results of Telionis and Werle.[8]

Velocity profiles in the stationary coordinate system, where the flow is unsteady, are shown in Fig. 1 for a fixed x station

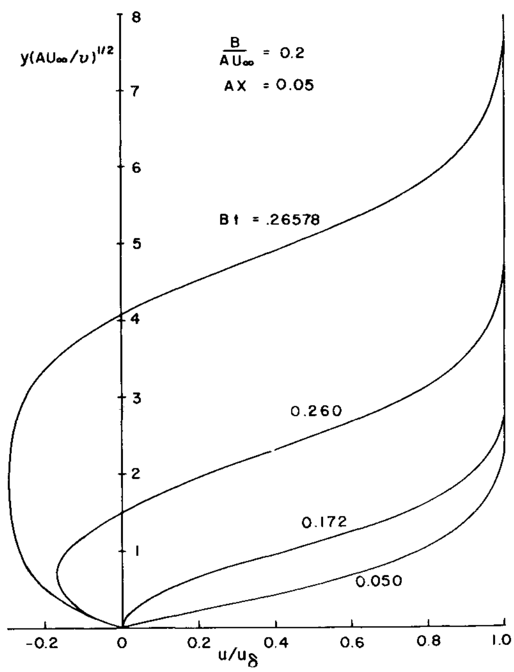

Fig. 1 Velocity profiles as "seen" in the fixed coordinate system.

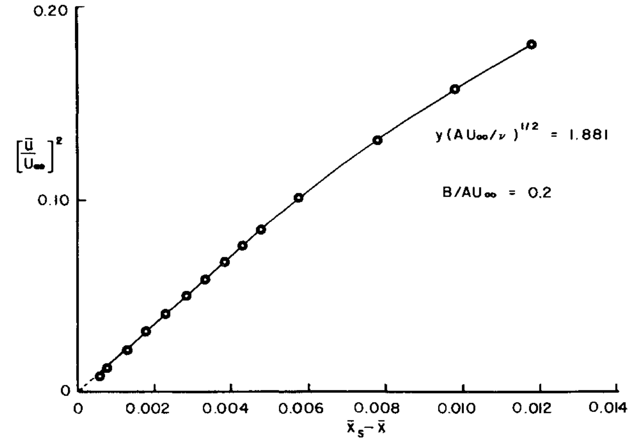

Fig. 2 Variation of the square of the $\bar{x}$ component near separation.

($Ax = 0.05$) for various values of the dimensionless time, Bt. As time increases and separation moves closer to this station, the velocity first becomes negative near the wall and then the region of reverse flow increases in size.

For this case, as the solution in the moving coordinate system approached $A\bar{x} \cong 0.3158$ the number of iterations increased rapidly and at $A\bar{x} = 0.3158$ it was not possible to obtain a converged solution. This was taken as an indication of a singularity in the vicinity of this point. As this point is approached the velocity profiles in the moving coordinate system approach a velocity profile for which the velocity and the shear vanish simultaneously at a point within the boundary layer.

Figure 2 presents the variation of the square of the velocity, as seen in the moving coordinate system, as a function of the distance from the separation point at the fixed value of $y[y(AU_\infty/\nu)^{1/2} = 1.881]$ which corresponds to the minimum velocity in the last station for which convergence was obtained. It is clear that near separation the velocity approaches zero as $\bar{u}^2 \sim (\bar{x}_{sep} - \bar{x})$ indicating a Goldstein-type square root singularity as postulated by Moore[1] and Telionis.[6] Another indication of this singular behavior is given in Fig. 3 when the normalized y component of velocity $[v/(U_\infty \nu A)^{1/2}]$ is presented as a function of ξ for several values of $y(AU_\infty/\nu)^{1/2}$, the normalized y coordinate. In the vicinity of $y(AU_\infty/\nu)^{1/2} = 1.881$ and for values of $y(AU_\infty/\nu)^{1/2}$ greater than this the y component of velocity increases rapidly as separation is approached, as one would expect for a Goldstein-type singularity. Near the wall, however, the y component of velocity does not appear to behave in a singular manner, indicating again that the singular point lies away from the wall.

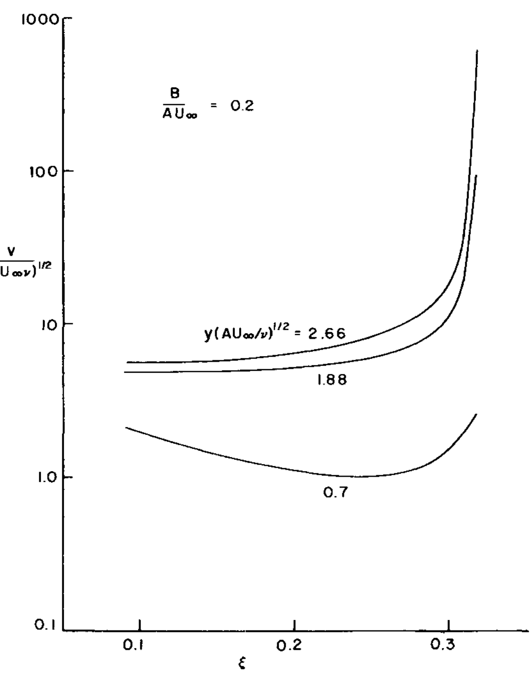

Fig. 3 Variation of the y component of velocity with ξ for several values of $y(AU_\infty/\nu)^{1/2}$.

The behavior of the flow indicated above verifies the Moore-Rott-Sears model for unsteady separation in that the separation point is characterized by: 1) the simultaneous vanishing of the shear and velocity at a point in the boundary layer in the flow ". . . seen by an observer moving with the separation point," and 2) a singular behavior of the boundary-layer equations at this point.

References

[1] Moore, F. K., "On the Separation of the Unsteady Laminar Boundary Layer," in *Boundary Layer Research*, edited by H. G. Görtler, Springer-Verlag, Berlin, 1958, pp. 296–310.

[2] Rott, N., "Unsteady Flow in the Vicinity of a Stagnation Point," *Quarterly of Applied Mathematics*, Vol. 13, 1956, pp. 444–451.

[3] Sears, W. R., "Some Recent Developments in Airfoil Theory," *Journal of the Aeronautical Sciences*, Vol. 23, No. 5, 1956, pp. 490–499.

[4] Rott, N., "Theory of Time-Dependant Laminar Flows," in *Theory of Laminar Flows*, edited by F. K. Moore, Princeton Univ. Press, Princeton, N.J., 1964, p. 432.

[5] Sears, W. R. and Telionis, D. P., "Unsteady Boundary Layer Separation," in *Fluid Dynamics of Unsteady, Three-Dimensional and Separated Flows*, Proceedings of a SQUID Workshop, edited by F. J. Marshall, Atlanta, Ga., June 1971.

[6] Telionis, D. P., "Boundary Layer Separation," Ph.D. thesis, Sept. 1970, Cornell University, Ithaca, N.Y.

[7] Blottner, F. G., "Finite-Difference Methods of Solution of the Boundary Layer Equations," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 193–205.

[8] Telionis, D. P. and Werle, M. J., "Boundary Layer Separation from Downstream Moving Boundaries," *Journal of Applied Mechanics*, Vol. 95, June 1973, pp. 369–374.

Burning Constant-Stoichiometric Ratio Relation—Some Clarifications

H. S. Mukunda* and B. N. Raghunandan†
Indian Institute of Science, Bangalore, India

Essenhigh mentions in his paper[1] with reference to burning of polymer spheres that a correlation between the burning constant (or evaporation constant) and oxidizer to fuel ratio can be found. This point has been reasserted by Essenhigh and Dreier.[2] This Note is primarily intended to point out that such a correlation should not be expected.

The burning constant K is defined by

$$t = K(d_o^2 - d^2) \quad (1)$$

where d_o = initial diameter, d = diameter at any time t. This correlation is inferred from an analysis of droplet combustion by neglecting the convection terms in the conservation relations. Such an analysis leads to

$$\frac{d_f}{d} = 1 + \frac{\rho_f \mathscr{D}_f}{\rho_o \mathscr{D}_o p_o} i \quad (2)$$

Received February 26, 1974.

Index categories: Combustion in Heterogeneous Media; Fuels and Propellants, Properties of.

* Lecturer, Department of Aeronautical Engineering.

† Research Student, Department of Aeronautical Engineering.